647510 **MSC1MatC101x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Jan/Feb.-2022 [NEW COURSE]
Algebra 1(Core (New))

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove First Isomorphism Theorem of groups.

(2) Prove that every group of order p^2 is Abelian, where p is a prime.

(3) Let H & K be subgroups of finite group $(G, \cdot)$ then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$.

Q.1 (b) Answer any ONE of the following questions. **[04]**

(1) Let H and G be groups and let $\phi: G \to H$ be a homomorphism. Show that $Ker\ \phi$ is a subgroup of G and $Im\ \phi$ is a subgroup of H.

(2) State and prove Cayley's theorem.

Q.2 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove first Sylow theorem.

(2) Prove that group of order 56 is not simple.

(3) Let G be a finite group and p be a prime such that $p^m | O(G)$ but $p^{m+1} \nmid O(G)$. Let n_p be the number of $sylow - p - subgroups$ of G. Then prove that $n_p | O(G)$.

Q.2 (b) Answer any ONE of the following questions. **[04]**

(1) Define the following terms:
a). Sylow-p-Subgroup b). Internal direct product

(2) Define Cyclic group and Abelian group. Prove that group of order 15 is cyclic.

Q.3 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Second Isomorphism Theorem of ring homomorphism.

(2) State and prove *Correspondence Theorem* of ring homomorphism.

(3) State and prove Fundamental Theorem of Ring theory.

Q.3 (b) Answer any ONE of the following questions. **[04]**

(1) Prove that in a nonzero commutative ring with unity, an ideal M is maximal ideal if and only if R/M is a field.

(2) Define the following terms:
a). Field b). Integral Domain c). Prime Ideal d). Maximal Ideal

Q.4 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Unique Factorization Theorem.

(2) State and prove Division Algorithm Theorem.

(3) Let $f(x) \in \mathbb{Z}[x]$ be primitive polynomial. Then prove that $f(x)$ is reducible over $\mathbb{Q}$ if and only if $f(x)$ is reducible over $\mathbb{Z}$.

Q.4 (b) Answer any ONE of the following questions. **[04]**

(1) Prove that product of two primitive polynomials is again a primitive polynomial.

(2) Define Irreducible Polynomial and prove that $x^3 - 5x + 10$ is irreducible over $\mathbb{Q}$.

Q.5 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Eisenstein Criterion.

(2) If R is a commutative ring with unity then prove that R is field if and only if R has precisely two ideals $\{0\}$ and R itself.

(3) State and prove Third isomorphism theorem of groups.

Q.5 (b) Answer any ONE of the following questions. **[04]**

(1) Define Subgroup and Normal Subgroup of a Group. Show that group of order 108 has a normal subgroup of order 27.

(2) Let $(\mathbb{Z}, +, \cdot)$ be the ring of integers and I be an ideal of $\mathbb{Z}$ then show that $I = k\mathbb{Z}$ for some $k \in \mathbb{N} \cup \{0\}$.

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